A METHOD OF TRANSVECTION IN THE ACTUAL COEFFICIENTS, AND AN APPLICATION TO EVECTANTS.

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The Evectant Operators E and $E^d I$.

Consider a form f and a mixed concomitant ψ,

$$f = a_0 x_1{}^n + n a_1 x_1{}^{n-1} x_2 + \ldots\ldots, \quad \psi = \xi^n x_1{}^n + n \xi^{n-1} \eta x_1{}^{n-1} x_2 + \ldots\ldots$$

Any invariant of f will still maintain its invariant properties if for f there be substituted $f + k\psi$. Let $I(a_0, a_1, \ldots\ldots)$ be the invariant of f. Then

$$I_1(a_0 + k\xi^n,\ a_1 + k\xi^{n-1}\eta,\ \ldots\ldots)$$

is also an invariant function (contravariant). Expanding, we obtain

$$I_1 = I + \sum_m \frac{k^m}{m!} \left(\xi^n \frac{\partial}{\partial a_0} + \xi^{n-1} \eta \frac{\partial}{\partial a_1} + \ldots\ldots \right)^m I.$$

Hence we have the invariant operator

$$E = \xi^n \frac{\partial}{\partial a_0} + \xi^{n-1} \eta \frac{\partial}{\partial a_1} + \ldots\ldots + \xi^n \frac{\partial}{\partial a_n} \equiv \sum_{s=0}^{s=n} \xi^{n-s} \eta^s \frac{\partial}{\partial a_s}$$

It has been shown that the most general form of invariant is

$$I = \binom{n}{\lambda^{(h)}} \binom{y}{\lambda^{(h-1)}} \ldots\ldots \binom{r}{\lambda}_{k_h - l'_h} I_{l''_h l'_h} I_{k_h - l''_h} \ldots\ldots {}_{k_1 - l'_1} I_{l''_1 l'_2} I_{k_1 - l''_1}\, a_{\lambda^{(h)}} a_{\mu^{(h-1)} + \lambda^{(h-1)}} \ldots\ldots$$
$$\ldots\ldots a_{\mu+\lambda}\, a_{r+\nu-\lambda}\,.$$

Hence the general form of contravariant is

$$EI = S_\lambda \left(a_{\mu^{(h-1)} + \lambda^{(h-1)}}\, a_{\mu^{(h-2} + \lambda^{(h-2)}} \ldots\ldots a_{r+\nu-\lambda} \xi^{n-\lambda^{(h)}} \eta^{\lambda^{(h)}} + \ldots\ldots + a_{\lambda^{(h)}} \ldots\ldots a_{\mu+\lambda} \right.$$
$$\left. \times \xi^{n+\lambda-r-\nu} \eta^{r+\nu-\lambda} \right).$$

Then, since $\partial/\partial x_1$ and $\partial/\partial x_2$ are contragredient to x_1, x_2, these may replace ξ and η in EI. Therefore,

$$E^d I = S_\lambda \left(a_{\mu^{(h-1)} + \lambda^{(h-1)}} \ldots\ldots a_{\mu+\lambda}\, a_{r+\nu-\lambda} \frac{\partial^n}{\partial x_1{}^{n-\lambda^{(h)}}\, \partial x_2{}^{\lambda^{(h)}}} + \ldots\ldots \right.$$

$$+a_{\lambda^{(h)}}\, a_{\mu^{(h-1)}+\lambda^{(h-1)}} \ldots\ldots a_{\mu+\lambda} \frac{\partial n}{\partial x_1^{\,n-r-\nu+\lambda}\, \partial x_2^{\,r+\nu-\lambda}})$$

is an invariant operator

Let $f=a_x^4=\binom{4}{\lambda}a_\lambda\, x_1^{4-\lambda}\, x_2^{\lambda}$.

$$\therefore H=(f,f)_2=\binom{2}{\lambda}\sum_{j=0}^{j=4} {}_{i_1}I_{i_2\, j-i_1}\, I_{j-i_2}\, a_{\mu+\lambda}\, a_{2+\nu-\lambda}\, x_1^{4-j}\, x_2^{j},$$

$$T=(f,(f,f)_3)_1=\binom{1}{\lambda'}\binom{2}{\lambda}\sum_{j'=0}^{j'=6} {}_{i'_1}I_{i'_2\, j'-i'_1}\, \dot{I}_{j'-i'_2;\; k_1-l'_1}\, I_{l''_1 l'_1}\, I_{k_1-l''_1}$$

$$\times a_{\mu'+\lambda'}\, a_{\mu+\lambda}\, a_{2+\nu-\lambda}\, x_1^{6-j'}\, x_2^{j'},$$

$$\mathbf{i}=(f,f)_4=\binom{4}{\lambda}a_\lambda\, a_{4-\lambda},$$

$$J=(f,(f,f)_2)_4=\binom{4}{\lambda'}\binom{2}{\lambda}{}_{k_1-l'_1}I_{l''_1 l'_1}I_{k_1-l''_1}\, a_{\lambda'}\, a_{\mu+\lambda}\, a_{2+\nu-\lambda}\, ,$$

$$E\mathbf{i}=\binom{4}{\lambda}(a_{4-\lambda}\xi^{4-\lambda}\eta^{\lambda}+a_\lambda\, \xi^\lambda\, \eta^{4-\lambda})=2[(a_4\xi^4+a_0\eta^4)-4(a_3\xi^3\eta+a_1\xi\eta^3)$$

$$+3a_2\xi^2\eta^2],$$

$$EJ=S_\lambda\,(a_{\mu+\lambda}\, a_{2+\nu-\lambda}\, \xi^{4-\lambda'}\eta^{\lambda'}+a_{\lambda'}\, a_{2+\nu-\lambda}\, \xi^{4-\mu-\lambda}\, \eta^{\mu+\lambda}+a_{\lambda'}\, a_{\mu+\lambda}\, \xi^{2+\lambda-\nu}\, \eta^{2+\nu-\lambda})$$

$$=6[(a_2a_4-a_3^2)\xi^4+2(\dot{a}_2a_3-a_1a_4)\xi^3\eta+(a_0a_4+2a_1a_3-3a_2^2)\xi^2\eta^2$$

$$+2(a_1a_2-a_0a_3)\xi\eta^3+(a_0a_2-a_1^2)\eta^4],$$

$$E^d\,\mathbf{i}=\binom{4}{\lambda}\left(a_{4-\lambda}\frac{\partial^4}{\partial x_1^{4-\lambda}\,\partial x_2^{\lambda}}+a_\lambda\,\frac{\partial^4}{\partial x_1^{\lambda}\,\partial x_2^{4-\lambda}}\right),$$

$$E^d\,J=S_\lambda\,(a_{\mu+\lambda}\, a_{2+\nu-\lambda}\frac{\partial^4}{\partial x_1^{4-\lambda'}\,\partial x_2^{\lambda'}}+a_{\lambda'}\, a_{2+\nu-\lambda}\frac{\partial^4}{\partial x_1^{4-\mu-\lambda}\partial v_2^{\mu+\lambda}}$$

$$+a_{\lambda'}\, a_{\mu+\lambda}\frac{\partial^4}{\partial x_1^{2+\lambda-\nu}\,\partial x_2^{2+\nu-\lambda}}).$$

Operating upon f, T, and H, we have,

$$(E^d\,\mathbf{i})f=(4-\lambda)\,!\lambda\,!\binom{4}{\lambda}\binom{4}{\lambda'}(2a_{4-\lambda}\, a_\lambda)=48i,$$

$$(E^d\,J)f=(4-\lambda)\,!\;\lambda\,!S_\lambda\,\binom{4}{\lambda}(3a_{\lambda'}\, a_{\mu+\lambda}\, a_{2+\nu-\lambda})=72J,$$

$$(E^d\,i)H=-2\binom{4}{\lambda}S_\lambda\, a_{\mu+\lambda}\, a_{2+\nu-\lambda}\, a_{4-j}=-48J,$$

$$(E^d J)H = 3S_\lambda\, S_{\lambda'}\, a_{\mu+\lambda}\, a_{2+v+\lambda}\, a_{\mu'-\lambda'} a_{2+v'-\lambda'} = 36[\tbinom{4}{\lambda} a_\lambda\, a_{4-\lambda}]^2 = 36 i^2,$$

$$(E^d i)T = 2\tbinom{4}{\lambda} S_\lambda\, a_{\mu'+\lambda'}\, a_{\mu+\lambda}\, a_{2+v-\lambda}\, a_{4-\lambda}\, x_1^{2-j'+\lambda}\, x_2^{j'-\lambda} \equiv 0,$$

$$(E^d J)T = 3S_\lambda\, S_{\lambda'}\, a_{\mu'+\lambda'}\, a_{\mu+\lambda}\, a_{2+v-\lambda}\, a_{\mu+\lambda}\, a_{2+v-\lambda}\, x_1^{2+\lambda-j'}\, x_2^{j'-\lambda'} \equiv 0.$$

In general, we have the theorem:

If I is an invariant of degree $(h+2)$ of an n^{ic}, $f=0$, then the following relations exist:

$$(E^d I)f = (h+2)\; n!\; I,$$

$$[E^d\,(f, f)_n](f\,(f, f)_r)_s = k(f, (f, (f, f)_r)_s)_n \quad (n \text{ even}),$$

$$[E^d\,(f, (f, f)_t)_{2(n-t)}](f\,(f, f)_r)_s = k[(f, f)_t, (f, (f, f)_r)_s)_n] \quad (n \text{ even}).$$

We next prove the following theorem:

*The nth transvectant of a form over an evectant of an invariant of the form is a numerical multiple of the invariant.**

In proof, consider an invariant

$$I = S_\lambda\,(a_{\lambda^{(h)}}\, a_{\mu^{(h-1)}+\lambda^{(h-1)}} \ldots\ldots a_{\mu+\lambda} a_{r+v-\lambda}).$$

Then $EI = S_\lambda\,(a_{\mu^{(h-1)}+\lambda^{(h-1)}} \ldots\ldots a_{\mu+\lambda} a_{r+v-\lambda}\, x_1^{n-\lambda^{(h)}}\, x_2^{\lambda^{(h)}} + a_{\lambda^{(h)}} a_{\mu^{(h-2)}+\lambda^{(h-1)}} \ldots\ldots$

$$\ldots\ldots a_{r+v-\lambda}\; x_1^{n-\mu^{(h-1)}-\lambda^{(h-1)}}\, x_2^{\mu^{(h-1)}+\lambda^{(h-1)}} + \ldots\ldots).$$

This is the sum of $(h+2)$ covariants, each of order n and degree $(h+1)$. The effect of taking the nth transvectant of f over EI is to introduce the multiplier $\binom{n}{t}$ and the factor a_t, [see (3)]. It is evident moreover that in the first covariant t may be replaced by $\lambda^{(h)}$, in the second by $(\mu^{(h-1)} + \lambda^{(h-1)})$, etc. Hence the result is a sum of $(h+2)$ invariants, each equal to I. Therefore

$$(f, EI)_n = (h+2)I, \text{ or } (E^d I)f = n!(f, EI)_n.$$

The System for the Binary $7-ic$.

The complete system of concomitants of the binary septic has been exhibited by Von Gall in *Math. Annalen*, Bd. 31, in symbolic form. Following is the derivation of the base covariants, in actual coefficients, of degree one to eight, and the invariants of degrees four and twelve. The set contains covariants of all orders from one to eleven inclusive, and is derived as an application of the previous theory.

Degree 1.

$f = a_x^7 = a_0 x_1^7 + 7a_1 x_1^6 x_2 + \ldots\ldots + a_7 x_2^7$; the form itself.

*Gordan, *Invariantentheorie.*

Degree 2.

H_x^{10}, the Hessian of f. Its seminvariant leading coefficient is $2(a_0a_2-a_1^2)$.

K_x^6, the fourth transvectant of f over itself. Its seminvariant leader is $2(a_0a_4-4a_1a_3+3a_2^2)$.

l_x^2, the sixth transvectant of f over itself. Its seminvariant leader is $2(a_0a_6-6a_1a_5+15a_2a_4-10a_3^2)$.

Degree 3.

γ_x^{11}, the Jacobian of f and K, with seminvariant leader $\frac{2}{5}a_0^2a_6-\frac{12}{5}a_0a_1a_5+6a_1^2a_4-12a_1a_2a_3+2a_0a_3^2+6a_2^3$.

r_x^3, the fifth transvectant of f and K. Seminvariant leading coefficient: $2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3$.

T_x^{15}, the Jacobian of f and H. Seminvariant leader: $a_6^2a_3-3a_0a_1a_2+2a_1^3$.

Degree 4.

$\triangle_x^8$, the Hessian of K. Seminvariant leader: $\frac{8}{5}a_0^2a_4a_6-8a_0a_2a_4^2+8a_0a_3^2a_4-\frac{32}{5}a_0a_1a_3a_6+\frac{32}{5}a_1^2a_3a_5-32a_1a_3^3+\frac{24}{5}a_0a_2^2a_6-\frac{24}{5}a_1a_2^2a_5-24a_2^3a_4-2a_0^2a_5^2-18a_1^2a_4^2-8a_0a_2a_3a_5+56a_1a_2a_3a_4+\frac{52}{5}a_0a_1a_4a_5+16a_2^2a_3^2$.

p_x^4, the fourth transvectant of K over itself. Seminvariant leading term: $2(2a_0a_1a_4a_7-\frac{84}{15}a_0a_2a_4a_6-10a_0a_3a_4a_5+4a_0a_4^3-\frac{48}{15}a_1^2a_3a_7-\frac{12}{15}a_1a_2a_3a_6+\frac{168}{15}a_1a_3^2a_5+\frac{36}{15}a_1a_2^2a_7-\frac{36}{15}a_2^3a_6+12a_2^2a_4^2-\frac{6}{15}a_0^2a_5a_7-\frac{74}{25}a_0a_1a_5a_6+\frac{126}{15}a_0a_2a_5^2+6a_1^2a_4a_6+2a_1a_3a_4^2+\frac{72}{15}a_2^2a_3a_5-\frac{12}{15}a_0a_2a_3a_7-36a_2a_3^2a_4+\frac{108}{225}a_0^2a_6^2+\frac{108}{225}a_1^2a_5^2+\frac{72}{15}a_0a_3^2a_6-\frac{306}{15}a_1a_2a_4a_5+12a_3^4)$.

s_x^{14}, the Jacobian of H and K. Seminvariant leading coefficient: $2(a_0^2a_2a_5-2a_0a_1a_2a_4-a_0a_1^2a_5-6a_1^2a_2a_3-a_0^2a_3a_4+4a_0a_1a_3^2-a_0a_2^2a_3+3a_1a_2^3+3a_1^3a_4)$.

Degree 5.

η_x^7, the Jacobian of r and K, with seminvariant leader: $2a_0^2a_2a_5a_7-7a_0^2a_3a_5a_6-5a_0^2a_4a_5^2-2a_0a_1^2a_5a_7+7a_0a_1a_2a_5a_6+62a_0a_1a_3a_5^2-7a_0a_1a_4^2a_5-57a_0a_2^2a_5^2+57a_0a_2a_3a_4a_5-20a_0a_3^3a_5-4a_0a_1a_2a_4a_7-47a_0a_1a_3a_4a_6-15a_0a_1a_4^2a_5+6a_1^3a_4a_7-21a_1^2a_2a_4a_6-138a_1^2a_3a_4a_5+75a_1^2a_4^3+135a_1a_2^2a_4a_5-105a_1a_2a_3a_4^2+60a_1a_3^3a_4-2a_0a_2^2a_3a_7-14a_0a_2a_3^2a_6-12a_1^2a_2a_3a_7-78a_1a_2^2a_3a_6+36a_1a_2a_3^2a_5-54a_2^3a_3a_5+120a_2^2a_3^2a_4-40a_2a_3^4+44a_0a_2^2a_4a_6-20a_0a_2a_4^3+10a_0a_3^2a_4^2-2a_0^2a_3a_4a_7+12a_0^2a_4^2a_6+80a_1^2a_3^2a_6-40a_1a_3^3a_4+8a_0a_1a_3^2a_7+6a_1a_2^3a_7+24a_2^4a_6+6a_2^3a_3a_5-60a_2^3a_3a_5$.

g_x^5, second transvectant of K over r. Seminvariant leading coefficient: $(a_0a_4-4a_1a_3+3a_2^2)(12a_1a_3a_7-20a_1a_4a_6+8a_1a_5^2+18a_2a_3a_6-10a_2^2a_7+2a_2a_4a_5-20a_3^2a_5+10a_3a_4^2-2a_0a_4a_7+2a_0a_5a_6)-\frac{2}{3}(a_0a_5-3a_1a_4+2a_2a_3)(30a_1a_3a_6-3a_1a_2a_7-27a_1a_4a_5-12a_2^2a_6-3a_2a_3a_5+30a_2a_4^2-15a_3^2a_4+3a_0a_3a_7-18a_0a_4a_6+15a_0a_5^2)+\frac{2}{5}(a_0a_6-a_1a_5-5a_2a_4+5a_3^2)(2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3)$.

Degree 6.

τ_x^2, the Hessian of r, with seminvariant leader: $2[(2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3)(6a_1a_3a_7-10a_1a_4a_6+4a_1a_5^2+9a_2a_3a_6-5a_2^2a_7+a_2a_4a_5-10a_3^2a_5+5a_3a_4^2-a_0a_4a_7+a_0a_5a_6)-(10a_1a_3a_6-a_1a_2a_7-9a_1a_4a_5-4a_2^2a_6-a_2a_3a_5+10a_2a_4^2-5a_3^2a_4+a_0a_3a_7-6a_0a_4a_6+5a_0a_5^2)^2]$.

β_x^8, the Jacobian of K and p. Seminvariant leader: $2(a_0a_4-4a_1a_3+3a_2^2)(\frac{8}{5}a_0a_2a_4a_7-\frac{48}{10}a_0a_3a_4a_6-2a_0a_4^2a_5-\frac{156}{15}a_1a_2a_3a_7+26a_1a_3^2a_6-\frac{528}{15}a_1a_3a_4a_5+6a_2^3a_7-\frac{234}{15}a_2^2a_3a_6+\frac{306}{15}a_2^2a_4a_5-\frac{96}{75}a_0a_1a_5a_7-\frac{36}{75}a_0a_2a_5a_6+6a_0a_3a_5^2+\frac{18}{5}a_1^2a_4a_7+18a_1a_4^3-\frac{28}{5}a_1a_2a_4a_6+\frac{54}{15}a_2a_3^2a_5-\frac{270}{15}a_2a_3a_4^2+\frac{12}{150}a_0^2a_6a_7-\frac{6}{15}a_0a_2a_4a_7+\frac{6}{15}a_0a_3^2a_7+\frac{6}{15}a_0a_1a_6^2-\frac{6}{15}a_1^2a_5a_6+\frac{252}{150}a_1a_2a_5^2+6a_3^3a_4)-2(a_0a_5-3a_1a_4+2a_2a_3)(2a_0a_1a_4a_7-\frac{84}{15}a_0a_2a_4a_6-10a_0a_3a_4a_5+4a_0a_4^3-\frac{16}{5}a_1^2a_3a_7-\frac{12}{15}a_1a_2a_3a_6+\frac{168}{15}a_1a_3^2a_5+\frac{36}{15}a_1a_2^2a_7-\frac{36}{15}a_2^3a_6+24a_2^2a_4^2-\frac{6}{15}a_0^2a_5a_7-\frac{74}{25}a_0a_1a_5a_6+\frac{126}{15}a_0a_2a_5^2+6a_1^2a_4a_6+2a_1a_3a_4^2+\frac{72}{15}a_2^2a_3a_5-\frac{1}{15}a_0a_2a_3a_7-36a_2a_3^2a_4+\frac{108}{225}a_0^2a_6^2+\frac{108}{225}a_1^2a_5^2+\frac{72}{15}a_0a_3^2a_6+\frac{306}{15}a_1a_2a_4a_5+12a_3^4)$.

Degree 7.

ϑ_x^5 is the Jacobian of p and r. Seminvariant leader: $2(2a_0a_1a_4a_7-\frac{28}{5}a_0a_2a_4a_6-10a_0a_3a_4a_5+4a_0a_4^3-\frac{16}{5}a_1^2a_3a_7-\frac{4}{5}a_1a_2a_3a_6+\frac{56}{5}a_1a_3^2a_5+\frac{12}{5}a_1a_2^2a_7-\frac{12}{5}a_2^3a_6+24a_2^2a_4^2-\frac{2}{5}a_0^2a_5a_7-\frac{74}{25}a_0a_1a_5a_6+\frac{42}{5}a_0a_2a_5^2+6a_1^2a_4a_6+2a_1a_3a_4^2+\frac{24}{5}a_2^2a_3a_5-\frac{4}{5}a_0a_2a_3a_7-36a_2a_3^2a_4+\frac{12}{25}a_0^2a_6^2+\frac{12}{25}a_1^2a_5^2+\frac{24}{5}a_0a_3^2a_6+\frac{102}{5}a_1a_2a_4a_5+12a_3^4)(10a_1a_3a_6-a_1a_2a_7-9a_1a_4a_5-4a_2^2a_6-a_2a_3a_5+10a_2a_4^2-5a_3^2a_4+a_0a_3a_7-6a_0a_4a_6+5a_0a_5^2)-(\frac{8}{5}a_0a_2a_4a_7-\frac{24}{5}a_0a_3a_4a_6-2a_0a_4^2a_5-\frac{52}{5}a_1a_2a_3a_7+26a_1a_3\ a_6-\frac{176}{5}a_1a_3a_4a_5+6a_2^3a_7-\frac{78}{5}a_2^2a_3a_6+\frac{102}{5}a_2^2a_4a_5-\frac{32}{25}a_0a_1a_5a_7-\frac{12}{25}a_0a_2a_5a_6+6a_0a_3a_5^2+\frac{18}{5}a_1^2a_4a_7+18a_1a_4^3-\frac{28}{5}a_1a_2a_4a_6+\frac{18}{5}a_2a_3^2a_5-18a_2a_3a_4^2+\frac{2}{25}a_0^2a_6a_7-\frac{2}{5}a_0a_2a_4a_7+\frac{2}{5}a_0a_3^2a_7+\frac{2}{5}a_0a_1a_6^2-\frac{2}{5}a_1^2a_5a_6+\frac{42}{25}a_1a_2a_5^2+6a_3^3a_4)(2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3)$.

a_x, the third transvectant of p over r. Seminvariant leading coefficient: $p_0r_3-3p_1r_2+3p_2r_1-p_3r_0$. The quartic invariant of the $7-ic$ is the discriminant of l_x^2, viz:

$C=2(14a_0a_1a_6a_7-24a_0a_2a_6^2+60a_0a_3a_5a_6-40a_0a_6a_4^2-24a_1^2a_5a_7+234a_1a_2a_5a_6-360a_1a_3a_5^2+240a_1a_4^2a_5+60a_1a_2a_4a_7-360a_2^2a_4a_6+990a_2a_3a_4a_5-600a_2a_4^3-40a_1a_3^2a_7+240a_2a_3^2a_6-60a_3^3a_5+375a_3^2a_4^2-a_0^2a_7^2-25a_1^2a_6^2-81a_2^2a_5^2-18a_0a_2a_5a_7+10a_0a_3a_4a_7-50a_1a_3a_4a_6)$.

Index $=$ weight $=14$.

The invariant of degree 12 is the discriminant of τ_x^2, viz:

$R=4[(2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3)(6a_1a_3a_7-10a_1a_4a_6+4a_1a_5^2+9a_2a_3a_6-5a_2^2a_7+a_2a_4a_5-10a_3^2a_5+5a_3a_4^2-a_0a_4a_7+a_0a_5a_6)-(10a_1a_3a_6-1a_1a_2a_7-9a_1a_4a_5-4a_2^2a_6-a_2a_3a_5+10a_2a_4^2-5a_3^2a_4+a_0a_3a_7-6a_0a_4a_6+5a_0a_5^2)^2]\times[(10a_1a_3a_6-a_1a_2a_7-9a_1a_4a_5-4a_2^2a_6-a_2a_3a_5+10a_2a_4^2-5a_3^2a_4+a_0a_3a_7-6a_0a_4a_6+5a_0a_5^2)(7a_1a_4a_7-7a_1a_5a_6-5a_2a_3a_7-22a_2a_4a_6+27a_2a_5^2+25a_3^2a_6-45a_3a_4a_5-2a_0a_5a_7+2a_0a_6^2+20a_4^3)-(6a_1a_3a_7-10a_1a_4a_6+4a_1a_5^2+9a_2a_3a_6-5a_2^2a_7+a_2a_4a_5-10a_3^2a_5+5a_3a_4^2-a_0a_4a_7+a_0a_5a_6)^2]-[(2a_0a_2a_7-7a_0a_3a_6+5a_0a_4a_5-2a_1^2a_7+7a_1a_2a_6+22a_1a_3a_5-25a_1a_4^2-27a_2^2a_5+45a_2a_3a_4-20a_3^3)(7a_1a_4a_7-7a_1a_5a_6-5a_2a_3a_7-22a_2a_4a_6+27a_2a_5^2+25a_3^2a_6-45a_3a_4a_5-2a_0a_5a_7+2a_0a_6^2+20a_4^3)-(10a_1a_3a_6-a_1a_2a_7-9a_1a_4a_5-4a_2^2a_6-a_2a_3a_5+10a_2a_4^2-5a_3^2a_4+a_0a_3a_7-6a_0a_4a_6+5a_0a_5^2)(6a_1a_3a_7-10a_1a_4a_6+4a_1a_5^2+9a_2a_3a_6-5a_2^2a_7+a_2a_4a_5-10a_3^2a_5+5a_3a_4^2-a_0a_4a_7+a_0a_5a_6)]^2$

Index $=$ weight $=42$.